

Loading containers on double-stack cars: Multi-objective optimization models and solution algorithms for improved safety and reduced maintenance cost

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Abstract

To improve safety measures of loading containers on double-stack rail cars, this paper develops a multi-objective optimization model that focuses on a number of practical requirements such as the center-of-gravity height of a loaded car and load balance considerations. A lexicographic goal programming approach is then used to address different priorities for potentially conflicting objectives and constraints. To minimize the center-of-gravity height, a linear-fractional programming technique is adopted in this study to transform the corresponding generalized mixed integer fractional problem into a sequence of mixed integer linear subproblems. A tabu search algorithm is proposed to get the close-to-optimal solutions of the large-scale double-stack car loading problems. Based on a set of matching and assigning rules, this paper further presents a two-stage heuristic algorithm for solving large-scale double-stack car loading problems. A real-world case study is used to examine the efficiency and effectiveness of the proposed optimization and heuristic procedures.

Keywords: Double-stack container loading; Multiple-objective optimization; Lexicographic goal programming; Linear-fractional programming; Heuristic algorithms

1. Introduction

Containerization provides an efficient and economic means of transporting goods. The vast majority of manufactured commodities moving via international trade are shipped in ocean-going containers, and the rail-truck intermodal segment plays an important role for regions that rely heavily on and wish to engage in emerging global markets. In recent decades, the growing international trade volumes have created many challenges for railroad systems around the world. As many rail terminals currently are running at or near capacity, railroad industries have responded to these challenges with new technological improvements, such as, double-stack intermodal train services. Introduced in 1984 from U.S West Coast ports, the double-stack train service allows containers to be stacked two high (double-stacked) so that freight containers are shipped more efficiently.

Double-stack container trains have been running in many countries including in the United States, Canada, and Australia. One important consideration in the double-stack container rail service model is that centers of gravity of loaded double-stack cars are much higher than those of regular rail cars and are variable. This raises significant concerns on the stability and related operational safety of double-stack container trains.

The center of gravity is a single point through which all forces affecting the rail car act. The vertical location of the center of gravity is of particular importance when examining the rail car's lateral stability. As a rail car runs on curve lines, a lateral force is created at the wheels causing it to turn. This force is opposed by an equal and opposite force, called a centrifugal force, which drives the rail car toward the outside of the curve lines. To counteract the centrifugal force, rails are superelevated in corners to allow the train to transverse the curve lines faster. Nadal's formula shown in Equation (1) (APTA 2007, USDOT-FRA 2011), used in railway design, relates the downward force exerted by the train's wheels on the rail to the lateral force of the wheels flange against the face of the rail.

$$\frac{L}{V} = \frac{\tan(\delta) - \mu}{1 + \mu \tan(\delta)} \quad (1)$$

where L and V refer to the lateral and vertical forces acting upon the rail and wheel, δ is the angle made when the wheel flange is in contact with the rail face, and μ is the coefficient of friction between the wheel and the rail.

Typically in railway design, it is desirable to have twice as much downward force than lateral force. This design standard governs the speed at which the trains can transverse specific curves. For a rail car to remain in equilibrium when traversing a curve, the resultant of the weight and the centrifugal force must be within the track width as in Fig 1B. In this situation, the resultant of the weight and normal forces are directed toward the center of the track, so that the load is distributed equally on the two rails.

When a train traverses a superelevated curve at a speed that is greater than recommended, and or when the center of gravity of the rail car is excessively high, it can lead to an overbalanced situation as seen in Fig 1C. As the train runs on curve lines, the weight and centrifugal forces act together at the center of gravity resulting in the green resultant force. This resultant force is directed away from the center of the track toward the high rail. The L/V ratio on the high rail tends to increase with speed and may result in wheel climb or rollover (Dukkipati 1988, AREMA 2003, Holowaty 2004, Loumiet 2005).

When a train traverses a superelevated curve at a speed that is less than recommended, and or when the center of gravity of the rail car is excessively high, it can lead to an underbalanced situation as seen in Fig 1A. In this case, the centrifugal forces are insignificant and the resultant force is shown by the green lines for both low and high centers of gravity. The resultant force is directed toward the low rail, resulting in unloading of the high rail to a critical point where the entire vertical load is on the lower rail and the high wheels lift off. In summary, the higher the center of gravity is, the more critical the underbalanced situation can be (Dukkipati 1988, AREMA 2003, Holowaty 2004, Loumiet 2005).

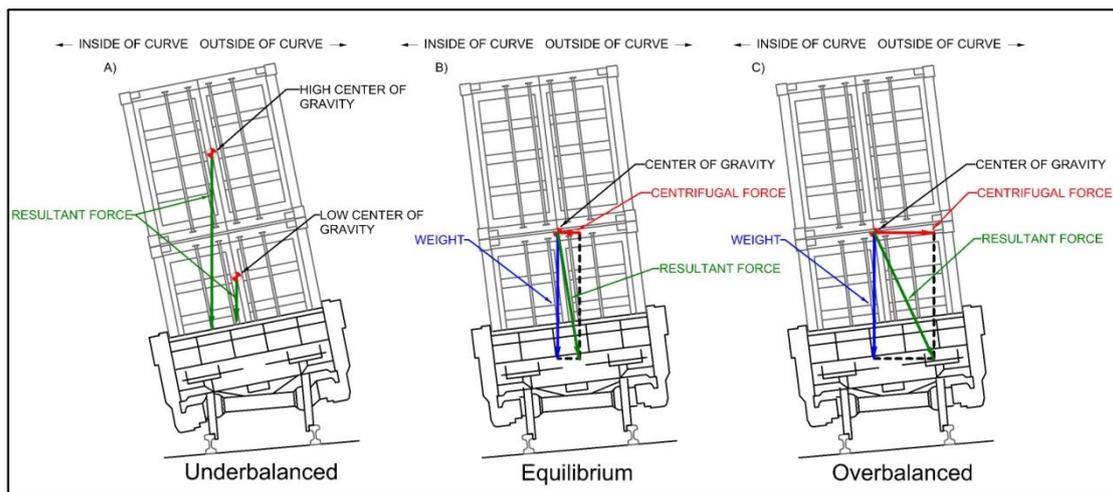


Figure 1. Underbalanced, Equilibrium, and Overbalanced train cars (adapted from Dukkipati 1988 and AREMA 2003)

The significance of cross-wind loading on rail cars leading to overturning has been extensively researched recently. When a train is exposed to cross-winds, there will be an aerodynamic loading on

the train creating a moment about the opposite rail. The weight of the rail car is the only stabilizing force, and if the train is running in a curve in an underbalanced situation, then the wind load will add to the overturning potential of the train. Rail cars with higher centers of gravity will be especially susceptible to the potential of overturning due to wind loads because the moment arm is increased, resulting in greater overturning forces (Gawthorpe 1994, RGS 1994, Schulte-Werning 1997, Fujii 1999, Lippert 1999, Lippert 2000, Matschke 2000, Andersson 2001, Cleon 2001, Brandbury 2003, Gautier 2003, Mancini 2003, Anderson 2004).

Just as the vertical location of the center of gravity of each rail car is of critical importance when considering the safety of the train, likewise, the location of the horizontal center of gravity for each rail car is also important. A concentration of loads on one end of a rail car tends to unload the wheels on the opposite end of the car. Although the weight of this type of load is typically insufficient to cause wheel lift by itself, when combined with the dynamic forces of train action, it will cause the car to be more susceptible to train dynamic forces which lead to derailment (Dukkipati 1988, Loumiet 2005). Our paper aims to systematically improve the important safety performance of feasible solutions that already satisfy the technical loading requirements, as any feasible solution can still have a very small probability of rail derailments which are associated with huge safety and operational impacts.

The safety issues associated with maintaining the lowest centers of gravity as possible on rail cars is clear. A less obvious, but equally important aspect of maintaining lower centers of gravity on rail cars, is reducing maintenance costs. There are so few rail derailments (relatively speaking) but the literature suggests that a lower center of gravity of rail cars lowers wear and tear and reduces maintenance costs. The ideal running condition for each train is in equilibrium. When a train is running out of equilibrium, excessive stresses occur on the trains and railways. While train overturning and/or derailment may not occur in each safety circumstance mentioned above, excessive car and track damage is occurring (AREMA 2003, Loumiet 2005).

The importance of providing rail cars with lower centers of gravity is embraced by industry as seen in the manufacturing and marketing of new rail cars. For example, Talgo America and Freight Car America have newer rail cars available that specifically tout their lower centers of gravity which provide increased safety and lower maintenance costs (Freight Car America Inc. 2011, Talgo Inc. 2011). Although the lower centers of gravity are nominal compared to other or previous models, the safety and maintenance implications are still credible. Double stack containers are more difficult to maintain lower centers of gravity due to the changing and random nature of loadings. Union Pacific Railroad, Norfolk Southern, and BNSF each have specific loading rules to follow when stacking double-stack rail cars (Norfolk Southern 1999, BNSF 2005, UP 2010). These rules, however, are simplistic and will not provide an optimization of vertical and horizontal centers of gravity. The U.S. Department of Transportation Federal Railroad Administration (FRA) reported that it is needed to develop and implement stricter guidelines for train make-up (weight loading). The current standards the railroad industry follows are potentially insufficient and the FRA can step in and encourage more conservative practices if it is deemed necessary (USDOT-FRA 2005).

To improve safety measures and reduce maintenance cost, this paper develops a multi-objective optimization model of double-stack rail car loading problem by taking the center-of-gravity heights and load balances of loaded cars into consideration. This study also aims to develop computationally efficient algorithms for solving the double-stack car loading problem. This paper is organized as follows. After this introductory Section and the literature review in Section 2, Section 3 describes detailed background information and some technical requirements of the double-stack railcar loading

problem. Section 4 presents a multi-objective optimization model for double-stack car loading. A lexicographic goal programming approach is then proposed in Section 5. This approach is able to handle the trade-offs between different and potentially conflicting objectives so that the developed models can be solved by using an optimization software package GAMS (Rosenthal, 2008). In order to optimize the height of the center of gravity, a linear-fractional technique is used to transform the generalized mixed integer linear-fractional programming problem into a sequence of mixed integer linear subproblems. Section 6 presents a tabu search algorithm for solving the double-stack railcar loading problem by generating an initial solution randomly and improving the solutions using 2-opt and tabu list techniques. In section 7, a two-stage heuristic algorithm is designed to solve the real-world instances according to a set of matching and assigning rules. Computational experiments are conducted in Section 8 to demonstrate the effectiveness and efficiency of the proposed algorithms.

2. Literature review

Research has been conducted into to the problem of optimizing the stacking of containers on rail cars. In an early study, Jähren and Rolle (1994) presented a computerized assignment algorithm for the double-stack railcar loading problem. They acknowledged that an assignment-type mathematical model could be developed, but priority-based heuristic solution algorithms were more suitable when precise information (e.g. about container weights and container destinations) was unavailable. Pacanovsky et al. (1995) described a prototype of a decision support system with different loading strategies related to the load factor, center of gravity, and uniformity in platform loads. Jähren et al. (1995) developed two automatic suggestion heuristic algorithms to improve the load quality of double-stack trains, namely a container-oriented method, and a location-oriented method. Essentially, these previous studies focus on heuristic algorithms of assigning containers to cars based on a set of loading strategies, but limited attention has been paid to the development of mathematical models for the double-stack car loading problem. In fact, there is a great need to construct and solve the optimization model, as an optimal assignment solution provides an exact benchmark for systematically testing various heuristic algorithms and alternative loading strategies.

The double-stack rail car loading problem is a variant of the Multiple Container Packing Problem (MCP) in which containers can be viewed as items and double-stack cars can be viewed as general containers. Additional constraints/objectives can be developed to include and optimize load balance and the center-of-gravity heights of loaded cars. The multiple containers packing problem is known to be an NP hard problem. When optimal solutions are unattainable for large-scale and complex instances, the heuristic approach is typically used to generate close-to-optimal solutions. For example, a genetic algorithm was presented by Raidl and Kodydek (1998), and an adaptive link adjustment evolutionary algorithm was developed by Soak et al. (2008).

The double-stack car loading problem has some structural similarities and inherent linkages with the container loading problems on cargo ships and in airplanes, which also aim to maximize the total payload within limited space. Imai et al. (2006) studied the stowage and loading plan for a container ship in which both the ship stability and the number of container re-handling are taken into account. In their study, the center of gravity was used as one of the safety constraints. Mongeau and Bes (2003) addressed the aircraft container loading problem in which a satisfactory distribution of the center of gravity of the aircraft was set as a hard constraint other than a (secondary) objective. From a multi-criteria optimization point of view, this paper specifically considers the center-of-gravity height

and the load balancing as the second and third objectives of the double-stack car loading problem.

3. Background information

To clearly describe loading patterns in the proposed mathematical model, this section starts with a background introduction to basic configurations of double-stack cars and containers, and then discusses the related technical requirements for loading containers on double-stack cars. Many detailed model inputs in this study are based on the specific considerations in China which are: (i) the payload of a car, (ii) the center-of-gravity height of a loaded car, and (iii) the load difference between two sets of loaded cars. Despite this model being specific to Chinese requirement, the proposed modeling methodology and solution algorithms can easily be adapted to the practice of other countries.

3.1. Basic configurations of double-stack cars

Double-stack rail cars are cars that allow containers to be stacked two high. These cars may be single, two, three, or five platform units that are articulated above shared trucks (more detailed descriptions can be found in the study by Jähren and Rolle, 1994). While the three or five-platform double-stack cars are popular in North America, due to the axle load limitation, double-stack cars in China are all manufactured with single platform.

There are two typical types of double-stack cars: Bulkhead and Interbox Connector (IBC). Bulkhead cars have risers at the ends of each platform that support the top container, while IBC cars rely on hand placed connectors between the bottom and top containers to lock the top container into place. With an IBC car, the weight of the top container is supported by the bottom container(s). IBC cars offer several advantages such as lower tare weights, higher load capacities, and the ability to load containers that are longer than 40 feet. Double-stack cars in China are all IBC cars, and their key parameters are shown in Table 1.

Table 1. Basic configurations of double-stack cars in China.

Maximal payload	Tare weight	Total length	Length of platform	Height of platform from rail surface	Height of center-of-gravity of empty car from the rail surface
78 tons	22 tons	19,466 mm	12,300 mm	290 mm	650 mm

Containers capable of being loaded on double-stack rail cars in China primarily include 20 ft and 40 ft International Organization for Standardization (ISO) containers.

3.2. Loading patterns of double-stack cars

To balance the loads of the two wheelsets of the double stack cars, the following two combinations are considered as infeasible in practice, i.e., two 20ft containers on the bottom and one 20 ft container on the top, and one 40ft container on the bottom and one 20 ft container on the top. So there are six possible combinations to load 20 ft ISO and 40 ft ISO containers onto double-stack cars:

- (1) two 20 ft containers on the bottom and one 40 ft container on the top shown in Figure 2;
- (2) only two 20 ft containers on the bottom shown in Figure 3;
- (3) only one 40 ft container on the bottom shown in Figure 4;
- (4) one 40 ft container on the bottom and another 40 ft container on the top;
- (5) one 40 ft container on the bottom and two 20 ft containers on the top; and
- (6) two 20 ft containers on the bottom and the other two 20 ft containers on the top.

To ensure the power collecting effect of electrified locomotives from the cable suspension wire on double-stack service lines, the height of the cable suspension wire should not exceed 6,330 mm, given the fixed heights of electrified locomotives. To avoid the electric shock to the double-stack trains, the maximal height of the loading-gauge (i.e. vertical clearance) of the electrified double-stack lines is 5,850 mm in China. As the height of a 40 ft container is 2,896 mm, and the height of the platform of a double-stack car is 290 mm. The 4th loading pattern of one 40 ft container on another 40 ft container leads to a total loading height of over 6,080 mm ($> 5,850$ mm), so this pattern is not allowed in China.

For the 5th loading pattern, the two 20 ft containers on the top are difficult to secure. When considering the maximal gross weight of a 20 ft container is 30.48 tons and the payload of a double-stack car is only 78 tons, loading pattern (6) gives an average load factor (or capacity factor) of 61.1% of each 20 ft container, which leads to an uneconomical option. As a result, there are only three permissible loading patterns of double-stack cars in China: (1) two 20 ft containers on the bottom, one 40 ft container on the top; (2) only two 20 ft containers on the bottom; and (3) only one 40 ft container on the bottom.

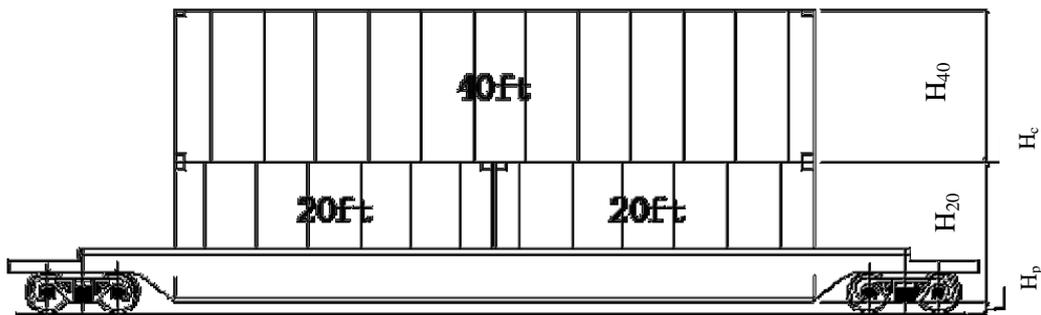


Figure 2. Loading pattern (1)



Figure 3. Loading pattern (2)

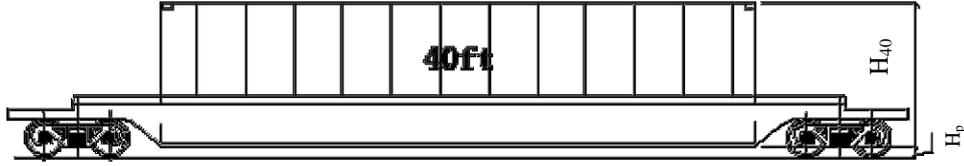


Figure 4. Loading pattern (3)

Accordingly, the following technical loading standards are currently used in China.

(1) The total gross weight of the containers loaded on a double-stack car should not exceed the maximal payload of the car, namely 78 tons.

(2) The center-of-gravity height of a loaded double-stack car should not exceed 2,400 mm.

(3) The weight difference between the two 20 ft containers on the bottom of a car should not exceed 10 tons.

4. Optimization model

The following notation is used to formulate the container loading problem on double-stack rail cars problem with center-of-gravity considerations.

4.1. Notations

(1) Indices/Constants/Parameters

i = 20 ft container index;

j = 40 ft container index;

k = double-stack car index;

I = total number of 20 ft containers;

J = total number of 40 ft containers;

K = total number of double-stack cars;

W_i^{20} = gross weight of the i^{th} 20 ft container, $i=1, \dots, I$;

W_j^{40} = gross weight of the j^{th} 40 ft container, $j=1, \dots, J$;

W_{car} = tare weight of a double-stack rail car;

H_p = height of the platform of the double-stack rail car from the rail surface;

H_c = height of the interbox connector between the top and the bottom containers, not including the two ends plugging into the corner fittings of the upper and lower containers. Generally, $H_c=30$ mm;

H_{20} = height of a 20 ft container;

H_{40} = height of a 40 ft container;

H_{car} = center-of-gravity height of an empty double-stack rail car from the rail surface;

\bar{P} = maximal payload of a double-stack car;

\bar{H} = maximal allowed center-of-gravity height of the loaded double-stack car;

\bar{W}_d = maximal allowed weight difference between the two 20 ft containers loaded on one

double-stack car;

M a very large positive number used in formulating if-then constraints.

(2) Variables

x_k^1 = 0-1 variable, if the k^{th} car uses loading pattern (1), $x_k^1=1$; otherwise, $x_k^1=0$

x_k^2 = 0-1 variable, if the k^{th} car uses loading pattern (2), $x_k^2=1$; otherwise, $x_k^2=0$

x_k^3 = 0-1 variable, if the k^{th} car uses loading pattern (3), $x_k^3=1$; otherwise, $x_k^3=0$

x_k^4 = 0-1 variable, if the k^{th} car is kept empty, $x_k^4=1$; otherwise, $x_k^4=0$

y_{ik} = 0-1 variable, if the i^{th} 20 ft container is loaded on the k^{th} car, $y_{ik}=1$; otherwise, $y_{ik}=0$

y_{ik}^A = 0-1 variable, if the i^{th} 20ft container is loaded on position A of the k^{th} car, $y_{ik}^A=1$;

otherwise, $y_{ik}^A=0$

y_{ik}^B = 0-1 variable, if the i^{th} 20 ft container is loaded on position B of the k^{th} car, $y_{ik}^B=1$;

otherwise, $y_{ik}^B=0$

z_{jk} = 0-1 variable, if the j^{th} 40 ft container is loaded on the k^{th} car, $z_{jk}=1$; otherwise $z_{jk}=0$

H = center-of-gravity height of a loaded car

For the convenience of mathematical modeling, we want to distinguish two different positions, namely A and B, on the bottom of a double-stack car when loading two 20 ft containers. The container on position A is assumed to be heavier than or has the same weight as that on position B. It should be remarked that, if two 20 ft containers are loaded on the bottom of a car, we must calculate the weight difference between the two 20 ft containers to satisfy the load balancing constraints and further consider the objective to minimize the weight difference of two 20 ft containers on one car. To mathematically model the weight difference, we introduce two different sets of variables for positions A and B, but the actual oppositions of two 20 ft containers on the bottom of a car can be exchanged without loss of generality.

Suppose the center of gravity of each container is located at its geometrical center under loading patterns (1-3), the corresponding center-of-gravity height of a loaded double-stack car can be expressed as Eqs. (2-4), respectively.

Pattern (1): two 20 ft containers on the bottom, one 40 ft container on the top.

$$H = \frac{W^{40} \cdot (H_p + H_{20} + H_c + H_{40} / 2) + (W_1^{20} + W_2^{20}) \cdot (H_p + H_{20} / 2) + W_{car} \cdot H_{car}}{W^{40} + W_1^{20} + W_2^{20} + W_{car}} \quad (2)$$

Specifically, for loading pattern (1), the center-of-gravity height of a loaded car is a weighted combination of three major components: (a) one 40 ft container with a weight of W^{40} and a

center-of-gravity height as $(H_p + H_{20} + H_c + H_{40}/2)$, (b) two 20 ft containers with a weight of $(W_1^{20} + W_2^{20})$ and a center-of-gravity height as $(H_p + H_{20}/2)$, and (c) an empty car with a weight of W_{car} and a center-of-gravity height as H_{car} .

Pattern (2): two 20 ft containers on the bottom

$$H = \frac{(W_1^{20} + W_2^{20}) \cdot (H_p + H_{20}/2) + W_{car} \cdot H_{car}}{W_1^{20} + W_2^{20} + W_{car}} \quad (3)$$

Pattern (3): one 40 ft container on the bottom

$$H = \frac{W^{40} \cdot (H_p + H_{40}/2) + W_{car} \cdot H_{car}}{W^{40} + W_{car}} \quad (4)$$

4.2. Problem Statement

Given a set of double-stack rail cars, a set of 20 ft containers and a set of 40 ft containers, the double-stack car loading problem under consideration aims to assign more 40 ft and 20 ft containers to the cars, and a container-car assignment should be either one of the above three feasible loading patterns.

In this study, we consider the following three objectives.

- (1) Maximize the TEUs loaded, where TEU stands for Twenty-foot Equivalent Unit, i.e., 1 TEU equals to one 20 ft container, then one 40 ft container equals to 2 TEUs.
- (2) Lower the center-of-gravity heights of loaded double-stack cars.
- (3) Balance the loads of the containers on double-stack cars.

To make the double-stack transport service more profitable, we set the first objective with the highest priority. As the center-of-gravity height is one of the most important factors regarding the operational safety of double-stack trains, this study assigns it as the second priority. The third objective of load balancing is considered as a secondary safety goal and it is assigned as the third priority. It should be remarked that, the corresponding hard constraints (originated from standard of practice) on the maximal payload, the maximal center-of-gravity height and the maximal weight difference between the two 20 ft containers on a car are included in the proposed optimization model.

4.3. Optimization model

(1) Objective functions

(a) Maximize **TEUs loaded**

$$Max \sum_{k=1}^K (\sum_{i=1}^I y_{ik} + 2 \cdot \sum_{j=1}^J z_{jk}) \quad (5)$$

Equation (5) aims to maximize the load factor of double-stack cars. Specifically, $\sum_{i=1}^I y_{ik}$ and

$\sum_{j=1}^J z_{jk}$ are the numbers of 20 ft containers and 40 ft containers, respectively, loaded on the k^{th} car. A

coefficient of 2 is used here to convert the number of 40 ft containers $\sum_{j=1}^J z_{jk}$ to the number of TEUs.

(b) Lower the **centers of gravity** of loaded cars

$$\text{Min Max}_{k=1}^K H_k \quad (6)$$

where H_k is the center-of-gravity height of the k^{th} car:

$$H_k = \frac{\sum_{j=1}^J (z_{jk} \cdot W_j^{40}) \cdot (H_p + H_{20} + H_c + H_{40}/2) + \sum_{i=1}^I (y_{ik} \cdot W_i^{20}) \cdot (H_p + H_{20}/2) + W_{car} \cdot H_{car} - \sum_{j=1}^J (z_{jk} \cdot W_j^{40}) \cdot (H_{20} + H_c) \cdot x_k^3}{\sum_{j=1}^J (z_{jk} \cdot W_j^{40}) + \sum_{i=1}^I (y_{ik} \cdot W_i^{20}) + W_{car}} \quad (7)$$

Equation (6) minimizes the maximal center-of-gravity height among all cars. In Equation (7), the fraction term corresponds to the center-of-gravity height of the k^{th} car under all loading patterns, extended from Eqs. (2-4) by adding the decision variables x_k^3 , y_{ik} and z_{jk} .

(c) Minimize the **weight difference** of 20 ft containers under loading patterns (1) and (2)

$$\text{Min Max}_{k=1}^K \left[\sum_{i=1}^I (y_{ik}^A \cdot W_i^{20}) - \sum_{i=1}^I (y_{ik}^B \cdot W_i^{20}) \right] \quad (8)$$

Equation (8) is intended to minimize the maximal weight difference of the two 20 ft containers on one car among all cars loaded with 20 ft containers. In this equation, $\sum_{i=1}^I (y_{ik}^A \cdot W_i^{20})$ and

$\sum_{i=1}^I (y_{ik}^B \cdot W_i^{20})$ represents the weight of the 20 ft container on position A and position B, respectively,

of the k^{th} car. By definition, the 20 ft container on position A is heavier than or has the same weight as that on position B, shown in Equation (19) later.

(2) Constraints

(a) **Matching constraints** (including Eqs. (9-16))

$$x_k^1 + x_k^2 + x_k^3 + x_k^4 = 1 \quad \forall \quad k = 1, \dots, K \quad (9)$$

Equation (9) ensures that one double-stack car can be only assigned to just one loading pattern or be kept empty.

$$\sum_{i=1}^I y_{ik} - 2x_k^1 - 2x_k^2 = 0 \quad \forall \quad k = 1, \dots, K \quad (10)$$

Equation (10) describes that if the loading pattern of a car is pattern (1) or (2) (i.e. $x_k^1=1$ or $x_k^2=1$),

there must be two 20 ft containers loaded on this car (i.e. $\sum_{i=1}^I y_{ik}=2$); otherwise, no 20 ft container is

loaded on this car.

$$\sum_{i=1}^I y_{ik}^A - x_k^1 - x_k^2 = 0 \quad \forall \quad k = 1, \dots, K \quad (11)$$

$$\sum_{i=1}^I y_{ik}^B - x_k^1 - x_k^2 = 0 \quad \forall \quad k = 1, \dots, K \quad (12)$$

Eqs. (10), (11) and (12) ensure that if the loading pattern of a car is (1) or (2) (i.e. $x_k^1=1$ or $x_k^2=1$),

there must be two 20 ft containers loaded on this car, one on position A (i.e. $\sum_{i=1}^I y_{ik}^A=1$) and the other

on position B (i.e. $\sum_{i=1}^I y_{ik}^B=1$); otherwise, there is no 20 ft container on this car.

$$\sum_{j=1}^J z_{jk} - x_k^1 - x_k^3 = 0 \quad \forall \quad k = 1, \dots, K \quad (13)$$

Equation (13) represents that if the loading pattern of a car is (1) or (3), there must be one 40 ft container loaded on this car ($\sum_{j=1}^J z_{jk}=1$); otherwise, no 40 ft container is loaded on this car.

$$y_{ik}^A + y_{ik}^B = y_{ik} \quad \forall \quad i = 1, \dots, I \quad k = 1, \dots, K \quad (14)$$

Equation (14) describes that a 20 ft container can either be loaded in position A or position B of one car or be left not loaded. More specifically, $y_{ik}=1$ corresponds to two mutually exclusive scenarios: (i)

$y_{ik}^A=1$ and $y_{ik}^B=0$, that is, the i^{th} 20 ft container is loaded on position A of the k^{th} car; (ii) $y_{ik}^A=0$ and

$y_{ik}^B=1$, so the i^{th} 20 ft container is loaded on position B of the k^{th} car. If $y_{ik}=0$, then $y_{ik}^A=y_{ik}^B=0$

which means the i^{th} 20ft container is not loaded on the k^{th} car.

$$\sum_{k=1}^K y_{ik} \leq 1 \quad \forall \quad i = 1, \dots, I \quad (15)$$

$$\sum_{k=1}^K z_{jk} \leq 1 \quad \forall \quad j = 1, \dots, J \quad (16)$$

Eqs. (15) and (16) represent that a 20 ft or 40 ft container can either be loaded to exactly one car or be left not loaded.

(b) Payload constraints

$$\sum_{i=1}^I (y_{ik} \cdot W_i^{20}) + \sum_{j=1}^J (z_{jk} \cdot W_j^{40}) \leq \bar{P} \quad \forall \quad k = 1, \dots, K \quad (17)$$

Equation (17) ensures that the total gross weight of the containers loaded in one double-stack car does not exceed the maximal payload of the car.

(c) Center-of-gravity height constraints

$$H_k \leq \bar{H} \quad \forall \quad k = 1, \dots, K \quad (18)$$

Equation (18) ensures that the center-of-gravity height of a loaded double-stack car does not exceed the allowed maximal value.

(d) Load balancing constraints

$$\sum_{i=1}^I (y_{ik}^A \cdot W_i^{20}) \geq \sum_{i=1}^I (y_{ik}^B \cdot W_i^{20}) \quad \forall \quad k = 1, \dots, K \quad (19)$$

Equation (19) is a definitional constraint for load balancing, which ensures that the weight of the 20 ft container on position *A* of a double-stack car is greater than or equal to that of the 20 ft container on position *B*.

$$\sum_{i=1}^I (y_{ik}^A \cdot W_i^{20}) - \sum_{i=1}^I (y_{ik}^B \cdot W_i^{20}) \leq \bar{W}_d \quad \forall \quad k = 1, \dots, K \quad (20)$$

Equation (20) ensures that the weight difference between the two 20 ft containers on the bottom of one car does not exceed the allowed maximal threshold.

(e) **Variable domain constraints**

$$x_k^1 \in \{0,1\} \quad \forall \quad k = 1, \dots, K$$

$$x_k^2 \in \{0,1\} \quad \forall \quad k = 1, \dots, K$$

$$x_k^3 \in \{0,1\} \quad \forall \quad k = 1, \dots, K$$

$$x_k^4 \in \{0,1\} \quad \forall \quad k = 1, \dots, K$$

$$y_{ik} \in \{0,1\} \quad \forall \quad i = 1, \dots, I \quad k = 1, \dots, K$$

$$y_{ik}^A \in \{0,1\} \quad \forall \quad i = 1, \dots, I \quad k = 1, \dots, K$$

$$y_{ik}^B \in \{0,1\} \quad \forall \quad i = 1, \dots, I \quad k = 1, \dots, K$$

$$z_{jk} \in \{0,1\} \quad \forall \quad j = 1, \dots, J \quad k = 1, \dots, K$$

5. Solution strategies

The proposed optimization model is difficult to solve using the standard linear/integer programming techniques, due to its following characteristics: 1) This multi-criteria problem has three objectives with different priorities, and 2) The objective and constraints regarding the center-of-gravity heights (Eqs. (6,7) and (18)) are non-linear fractional functions.

To reformulate constraint (18) as a linear function of assignment variables, one can move the denominator of H_K on the left hand side to the right hand side, but the term

$$\sum_{j=1}^J (z_{jk} \cdot W_j^{40}) \cdot (H_{20} + H_c) \cdot x_k^3$$

in the nominator of H_K is still a nonlinear function involving

variables x_k^3 and z_{jk} . From Figs. 2-4, it can be seen that the center-of-gravity heights under loading pattern (2) and (3) are much lower than those under loading pattern (1). For simplicity, we can only enforce the center-of-gravity height constraints under loading pattern (1), and omit those under pattern (2) and (3). As a result, Equation (18) is reformulated as Equation (21), in which we change

$-\sum_{j=1}^J (z_{jk} \cdot W_j^{40}) \cdot (H_{20} + H_c) \cdot x_k^3$ to $-M \cdot (1-x_k^1)$ so that, under patterns (2) or (3),

$-M \cdot (1-x_k^1)$ reduces to a large negative number $-M$ and the inequality is always valid. On the other hand, under pattern (1) ($x_k^1=1$), the above term reduces to 0, and the resulting equation means the center-of-gravity height of a loaded double-stack car under loading pattern (1) does not exceed the allowed maximal value \bar{H} .

$$\begin{aligned} & \sum_{j=1}^J (z_{jk} \cdot W_j^{40}) \cdot (H_p + H_{20} + H_c + H_{40} / 2) + \sum_{i=1}^I (y_{ik} \cdot W_i^{20}) \cdot (H_p + H_{20} / 2) + W_{car} \cdot H_{car} - M \cdot (1-x_k^1) \\ & \leq \sum_{j=1}^J (z_{jk} \cdot W_j^{40}) \cdot \bar{H} + \sum_{i=1}^I (y_{ik} \cdot W_i^{20}) \cdot \bar{H} + W_{car} \cdot \bar{H} \quad \forall k = 1, \dots, K \end{aligned} \quad (21)$$

At this point, the constraint (21) on the center-of-gravity height has been transformed to a linear inequality, but the objective function representing the center-of-gravity height (Equation (6)) is still a fractional function with variables being included in both nominator and denominator.

To address complexity of the proposed formulation, we propose the following solution strategies to decompose the double-stack car loading problem into a sequence of subproblems.

1) A lexicographic goal programming approach is first used to consider different priorities associated with three potentially conflicting objectives.

2) A linear-fractional programming technique is used to transform the original problem into iterative mixed integer linear problems so as to approximate the target center-of-gravity height in the optimal solution.

5.1. Lexicographic goal programming approach and model reformulation

As shown in Tamiz et al. (1998), goal programming models can be classified into two major categories. In the first category, the unwanted deviations are assigned with the weights corresponding to their relative importance to the decision maker, and the resulting total deviation is minimized as an Archimedian sum. In the second lexicographic approach, a sequence of minimization problems (corresponding to each priority) is solved, while each problem maintains the minimal objective values reached by the higher priority level(s). As the problem under consideration has three objectives, we will focus on those objective functions sequentially through the following three steps.

Step 1: Consider the first objective that maximizes the TEUs loaded by solving the following optimal problem:

$$\begin{aligned} & \text{Max} \sum_{k=1}^K \left(\sum_{i=1}^I y_{ik} + 2 \cdot \sum_{j=1}^J z_{jk} \right) \\ \text{(P1)} \quad & \text{s.t. Constraints (9)-(17), (19-21)} \\ & \text{Variable domain constraints} \end{aligned}$$

It can be verified that P1 is a mixed integer linear programming problem that can be easily solved by using standard optimization software packages, for example, GAMS. After solving P1, the maximum TEUs loaded can be obtained, denoted as TEU_{\max} .

Step 2: Consider the second objective Equation (6) that minimizes the maximal height of the center of gravities among all cars while maintaining the same TEUs loaded as TEU_{\max} .

This nonlinear objective function as shown in Equation (6) can be further simplified by only focusing on loading pattern (1). That is, we have the following objective function Equation (22), by

replacing $-\sum_{j=1}^J (z_{jk} \cdot W_j^{40}) \cdot (H_{20} + H_c) \cdot x_k^3$ by $-M \cdot (1-x_k^1)$.

$$\text{Min Max}_{k=1}^K \frac{\sum_{j=1}^J (z_{jk} \cdot W_j^{40}) \cdot (H_p + H_{20} + H_c + H_{40} / 2) + \sum_{i=1}^I (y_{ik} \cdot W_i^{20}) \cdot (H_p + H_{20} / 2) + W_{car} \cdot H_{car} - M \cdot (1-x_k^1)}{\sum_{j=1}^J (z_{jk} \cdot W_j^{40}) + \sum_{i=1}^I (y_{ik} \cdot W_i^{20}) + W_{car}} \quad (22)$$

We then set the maximal TEUs obtained from solving P1 as a new equality constraint (23).

$$\sum_{k=1}^K \left(\sum_{i=1}^I y_{ik} + 2 \cdot \sum_{j=1}^J z_{jk} \right) = TEU_{\max} \quad (23)$$

This leads to subproblem P2.

$$\begin{aligned} & \text{optimize} \quad Eq.(22) \\ (P2) \quad & \text{s.t.} \quad \text{Constraints} \quad (9)-(17),(19)-(21),(23) \\ & \quad \quad \text{Variable domain constraints} \end{aligned}$$

By solving P2, the minimum value of the maximum center-of-gravity height among all cars can be obtained, denoted as H_{\min} . This results in the following new inequality constraints for the next step.

$$\frac{\sum_{j=1}^J (z_{jk} \cdot W_j^{40}) \cdot (H_p + H_{20} + H_c + H_{40} / 2) + \sum_{i=1}^I (y_{ik} \cdot W_i^{20}) \cdot (H_p + H_{20} / 2) + W_{car} \cdot H_{car} - M \cdot (1-x_k^1)}{\sum_{j=1}^J (z_{jk} \cdot W_j^{40}) + \sum_{i=1}^I (y_{ik} \cdot W_i^{20}) + W_{car}} \leq H_{\min} \quad \forall k = 1, \dots, K \quad (24)$$

Step 3: Consider the third objective that minimizes the weight difference between the two 20 ft containers on one car among all cars by solving the following problem.

$$\begin{aligned} & \text{Min Max}_{k=1}^K \left(\sum_{i=1}^I (y_{ik}^A \cdot W_i^{20}) - \sum_{i=1}^I (y_{ik}^B \cdot W_i^{20}) \right) \\ (P3) \quad & \text{s.t.} \quad \text{Constraints} \quad (9)-(17),(19)-(21),(23),(24) \\ & \quad \quad \text{Variable domain constraints} \end{aligned}$$

5.2. Linear-fractional programming technique

Subproblem P2 is a generalized mixed integer fractional problem. The following discussion first aims to solve P2 as a non-fractional but still nonlinear subproblem P4, where one additional variable γ is introduced to translate the original linear fractional objective function Equation (22) of P2 into a non-fractional constraint (25).

$$\begin{aligned} & \text{Min } \gamma \\ (P4) \quad & \text{s.t.} \quad \text{Constraints} \quad (9)-(17), (19)-(21),(23),(25) \\ & \quad \quad \text{Variable domain constraints} \end{aligned}$$

where Equation (25) is expressed as

$$\begin{aligned} & \sum_{j=1}^J (z_{jk} \cdot W_j^{40}) \cdot (H_p + H_{20} + H_c + H_{40} / 2) + \sum_{i=1}^I (y_{ik} \cdot W_i^{20}) \cdot (H_p + H_{20} / 2) + W_{car} \cdot H_{car} - M \cdot (1-x_k^1) \\ & \leq \sum_{j=1}^J (z_{jk} \cdot W_j^{40}) \cdot \gamma + \sum_{i=1}^I (y_{ik} \cdot W_i^{20}) \cdot \gamma + W_{car} \cdot \gamma \quad \forall k = 1, \dots, K \end{aligned} \quad (25)$$

Eq. (25) is nonlinear because gamma is not a constant but a variable in this equation, which leads to multiplications of variables in this equation. To remove nonlinearity related to γ in Equation (25), we use the bisection algorithm (shown in Figure 5) to convert P4 to a sequence of mixed integer linear programming problems P5, in which γ is a fixed value and a newly introduced variable t in linear Equation (26) is used to test the feasibility of the subproblem for a given γ . According to the characteristics of the problem, we can set $l=650$ mm (height of center-of-gravity of empty car from the rail surface) and $u=2,400$ mm (maximal allowed center-of-gravity height of the loaded double-stack car) as the initial values in this bisection algorithm.

$$\begin{aligned} & \sum_{j=1}^J (z_{jk} \cdot W_j^{40}) \cdot (H_p + H_{20} + H_c + H_{40} / 2) + \sum_{i=1}^I (y_{ik} \cdot W_i^{20}) \cdot (H_p + H_{20} / 2) + W_{car} \cdot H_{car} - M \cdot (1-x_k^1) \\ & \leq \sum_{j=1}^J (z_{jk} \cdot W_j^{40}) \cdot \gamma + \sum_{i=1}^I (y_{ik} \cdot W_i^{20}) \cdot \gamma + W_{car} \cdot \gamma + t \quad \forall k = 1, \dots, K \end{aligned} \quad (26)$$

Min t

(P5) *s.t.* Constraints (9)-(17), (19)-(21),(23),(26)

Variable domain constraints

Note that, if the optimal solution of P5 satisfies $t^* \leq 0$, then the problem is feasible for a given γ ; otherwise, the problem is infeasible. It should be remarked that, all the constraints in P5 are linear functions.

Given: interval $[l, u]$ that contains optimal γ
repeat: solve feasibility problem P5 for $\gamma = (u+l)/2$
 if feasible $u := \gamma$; if infeasible $l := \gamma$
until $u-l \leq \epsilon$

Figure 5. The bisection algorithm for solving P4

6. A tabu search algorithm for double-stack car loading problem

Tabu search algorithm is a commonly used heuristic algorithm in combinational optimization. It is an improved iterative local search algorithm. Generally, the tabu search starts from an initial solution which can be generated randomly or be generated by other approaches. At each generation, the neighboring solutions of the current solution will be searched and evaluated, and the best solution among them becomes the new current solution (even sometimes its quality is worse than that of current solution). As opposed to general local search algorithms, tabu search algorithm can escape from local optimal solutions by allowing a degradation of the objective. To avoid the repetitive searching of the low-quality local optimal solutions, the local optimal solutions that were recently examined are forbidden and inserted into a constantly updated tabu list.

Ichoua et al. (2003) stated the process of the tabu search algorithm which was shown in Figure 6.

Set a randomly generated solution as the current solution.

While the stopping criteria are not met, do:

- Generate a number of non-tabu neighboring solutions of the current solution by using local search strategy and put them into the candidate solution set.
- Select the best solution from the candidate solution set and set it as the new current solution.
- If the current solution is better than the best known solution then set the current solution as the best known solution.
- Update the tabu list.

Figure 6. The process of tabu search algorithm

As a combinational optimization problem, the double-stack car loading problem can also be solved by the tabu search algorithm, and close-to-optimal solutions can be obtained. In our tabu search algorithm for the double-stack car loading problem, the following elements are designed.

(1) Representation of a solution. We use two lists of containers to represent a solution for the double-stack car loading problem, one list of 20 ft containers and another list of 40 ft containers. A solution can be decoded to a loading plan by loading the 20 ft containers and 40 ft containers sequentially on the cars according to the matching constraints of the problem. For example, for a problem with nine 20 ft containers, three 40 ft containers and four cars, the 20 ft container list (853924176) and 40 ft container list (312) can stand for a solution which corresponds to the following loading plan, to load 20 ft containers No. 8 and No. 5 together with 40 ft container No. 3 on car No. 1, to load 20 ft containers No. 3 and No. 9 together with 40 ft container No. 1 on car No. 2, to load 20 ft containers No. 2 and No. 4 together with 40 ft container No. 2 on car No. 3, to load 20 ft containers No. 1 and No. 7 on car No. 4. Cars No.1, 2 and 3 are loaded with pattern (1). Car No.4 is loaded with pattern (2).

Take another example to load eight 20 ft containers, five 40 ft containers on six cars, the 20 ft container list (68537241) and 40 ft container list (53142) can stand for a solution which corresponds to the following loading plan, to load 20 ft containers No. 6 and No. 8 together with 40 ft container No. 5 on car No. 1, to load 20 ft containers No. 5 and No. 3 together with 40 ft container No. 3 on car No. 2, to load 20 ft containers No. 7 and No. 2 together with 40 ft container No. 1 on car No. 3, to load 20 ft containers No. 4 and No. 1 together with 40 ft container No. 4 on car No. 4, to load 40 ft containers No. 2 on car No. 5. Cars No.1, 2, 3 and 4 are loaded with pattern (1). Car No.5 is loaded with pattern (3). Car No.6 is kept empty.

(2) Evaluation of a solution. We can evaluate a solution according to its correspondent loading plan. For a solution S to a problem with K cars, each car corresponds to a loading scheme. We can calculate the TEUs loaded, total gross weight of the containers loaded, the center of gravity height, and the weight difference of two 20 ft containers of each car. According to the payload constraints, the center of gravity height constraints and the load balancing constraints of the double-stack car loading problem, we can determine if the loading scheme of each car is feasible or not. For car k ($k=1, \dots, K$), we use F_k stands for its loading feasibility ($F_k=1$ means the loading scheme of car k is feasible, $F_k=0$ means the loading scheme of car k is infeasible). Suppose the TEUs loaded of car k is $TEUs_k$, the

center-of-gravity height of car k is H_k , the weight difference of two 20 ft containers of car k is W_k .

Then the three objective values of solution S , namely the TEUs loaded ($TEUs$), the maximal center-of-gravity height (H) and the maximal weight difference of the two 20 ft containers (W) can be calculated by Eqs (27), (28) and (29) respectively. Thus the quality of solution S can be determined by these three objective values.

$$TEUs = \sum_{k=1}^K (TEUs_k \cdot F_k) \quad (27)$$

$$H = \mathbf{Max}_{k=1}^K (H_k \cdot F_k) \quad (28)$$

$$W = \mathbf{Max}_{k=1}^K (W_k \cdot F_k) \quad (29)$$

(3) Local search strategy. We simply swap the positions of two randomly selected 20 ft containers and the positions of two randomly selected 40 ft containers in current solution to get a neighboring solution. For example, if the current solution is: 20 ft container list (e.g., 853924176) and 40 ft container list (e.g., 3124), suppose the randomly selected two 20 ft containers are 20 ft container No. 5 and No. 2, and the randomly selected two 40 ft containers are 40 ft container No. 3 and No. 4, by swapping the positions of the two 20 ft containers and the positions of the two 40 ft containers, we can get a neighboring solution: 20 ft container list (823954176) and 40 ft container list (4123).

(4) Determining of the candidate solution set. A certain number of randomly generated non-tabu neighboring solutions of the current solution will be put into the candidate solution set to be evaluated.

(5) Determining of the tabu solutions. The best solution in the candidate solution set that were recently examined will be added to the end of the tabu list, and the first solution on the tabu list will be released simultaneously. The length of the tabu list will be determined according to the scale of the problem.

(6) The termination criterion. The algorithm terminates after evolve a certain number of generations (iterations).

7. A two-stage heuristic algorithm for double-stack car loading problem

The double-stack cars are identical in China, so the core of the proposed container-car assignment problem reduces to the container pairing/matching problem. Otherwise, if the cars are not identical, the proposed model and algorithms should be modified to consider different types/parameters of the cars. Considering all the objectives of the double-stack car loading problem as mentioned before, we also develop a two-stage heuristic procedure as follows:

(I) Pairing, matching and assignment. Pair 20 ft containers, match 20 ft container pairs with 40 ft containers for loading pattern (1), and assign containers on double-stack cars to construct an initial solution;

(II) Improvement. Improve solution quality by switching container positions.

In the proposed lexicographic goal programming approach, we tackle the three objectives sequentially. In the above heuristic procedure, we first focus on loading pattern (1), because this pattern has the highest loading factor with 4 TEUs per car. Stage I is composed of the following three steps. The first step aims to pair 20 ft containers sequentially according to load balancing constraints and the third objective to minimize the weight difference between 20 ft containers on a car. The second step

focuses on generating a good initial matching combination to maximize the number of TEUs loaded (the first objective). It should be remarked that, in this process, we try to match a heavy 40 ft container with a pair of heavy 20 ft containers (subject to the payload constraint), which inexplicitly helps to lower the center of gravity (the second objective). In the third step of stage I, the matched and unmatched containers are assigned to cars according to a number of assigning rules relevant to the numbers of cars and containers. Stage II is intended to lower the center of gravity and minimize the weight difference between 20 ft containers on a car by slightly changing the initial solution.

Under loading pattern (1), the discussion below aims to describe the feasible region of 20 ft containers (in terms of their weights), given a 40 ft container with a gross weight of W^{40} . Suppose

the weights of two 20 ft containers are W_1^{20} and W_2^{20} respectively, where $W_1^{20} \geq W_2^{20}$.

The payload constraint (17) can be simplified to

$$W_1^{20} + W_2^{20} \leq \bar{P} - W^{40} \quad (30)$$

For loading pattern (1), the center-of-gravity height of car k can be calculated by Eq (2), i.e.

$$H = \frac{W^{40} \cdot (H_p + H_{20} + H_c + H_{40} / 2) + (W_1^{20} + W_2^{20}) \cdot (H_p + H_{20} / 2) + W_{car} \cdot H_{car}}{W^{40} + W_1^{20} + W_2^{20} + W_{car}}$$

With constraint (18), we get

$$H = \frac{W^{40} \cdot (H_p + H_{20} + H_c + H_{40} / 2) + (W_1^{20} + W_2^{20}) \cdot (H_p + H_{20} / 2) + W_{car} \cdot H_{car}}{W^{40} + W_1^{20} + W_2^{20} + W_{car}} \leq \bar{H}$$

By putting $W_1^{20} + W_2^{20}$ on the left hand side, the center-of-gravity height constraint (18) leads to

$$W_1^{20} + W_2^{20} \geq \frac{W^{40} (H_p + H_{20} + H_c + H_{40} / 2 - \bar{H}) - W_{car} (\bar{H} - H_{car})}{\bar{H} - H_p - H_{20} / 2} = aW^{40} - b \quad (31)$$

$$\text{where } a = \frac{H_p + H_{20} + H_c + H_{40} / 2 - \bar{H}}{\bar{H} - H_p - H_{20} / 2}; \quad b = \frac{W_{car} (\bar{H} - H_{car})}{\bar{H} - H_p - H_{20} / 2}.$$

The load balancing constraints (19-20) give

$$W_1^{20} - W_2^{20} \geq 0 \quad (32)$$

$$W_1^{20} - W_2^{20} \leq \bar{W}_d \quad (33)$$

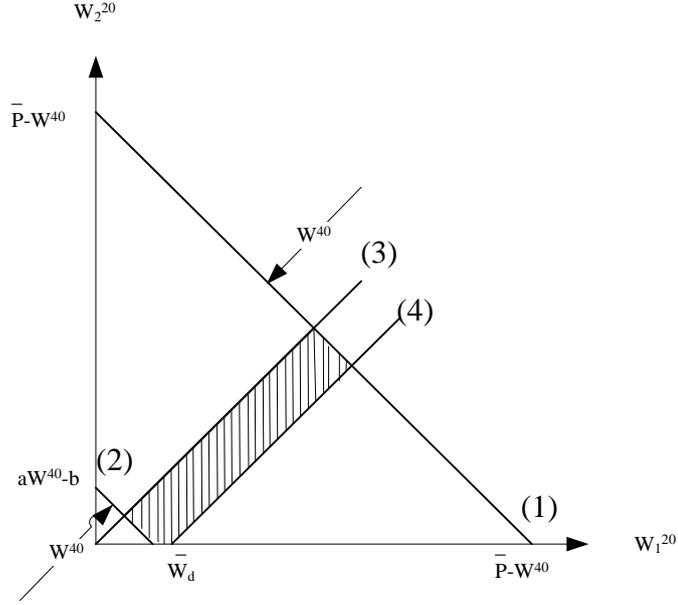


Figure 7. The feasible region of the weights of 20 ft containers which can be matched to a 40 ft container with the weight of W^{40} under loading pattern (1)

In Figure 7, Lines (1), (2), (3) and (4) correspond to Equations $W_1^{20} + W_2^{20} = \bar{P} - W^{40}$,

$W_1^{20} + W_2^{20} = aW^{40} - b$, $W_1^{20} - W_2^{20} = 0$ and $W_1^{20} - W_2^{20} = \bar{W}_d$ separately. Figure 7 shows

that, when W^{40} increases, the line (1) moves toward the origin of the coordinate while the line (2)

leaves away from the origin of the coordinate, then the feasible region of matching 20 ft containers (marked in the shaded area) shrinks accordingly. That is, the heavier a 40 ft container, the more difficult it is to find matching 20 ft containers. Therefore, in the matching process, our algorithm gives the priority for heavier 40 ft containers. Furthermore, when selecting the 20 ft container pair to be matched, we should select heavy ones to lower the overall center of gravity of the loaded double-stack car, subject to the payload constraints. This forms the basis for our sequential heuristic matching algorithm.

7.2. Matching 20 ft container pairs to 40 ft containers and assigning containers on cars

Some notations for the heuristic algorithm

N^{20} = total number of 20 ft container pairs;

n = index of 20 ft container pair index, $n=1, \dots, N^{20}$;

$Pair(n,1)$ = the No. of the first 20 ft container in the n^{th} pair;

$Pair(n,2)$ = the No. of the second 20 ft container in the n^{th} pair;

M = total number of matching groups;

m = index of matching groups (a matching group is a combination of a 20 ft container pair and a 40 ft container which can be loaded in one double-stack car), $m=1, \dots, M$;

$Match(m,1)$ = the No. of the first 20 ft container in the m^{th} matching group;

$Match(m,2)$ = the No. of the second 20 ft container in the m^{th} matching group;

$Match(m,3)$ = the No. of the 40 ft container in the m^{th} matching group;

U^{20} = total number of unmatched 20 ft container pairs;
 U^{40} = total number of unmatched 40 ft containers.

Step 0: Read input data as unordered 40 ft and 20 ft containers.

Step 1: Sorting

Sort 40 ft containers according to a descending order of their weights, i.e.,

$$W_j^{40} \geq W_{j+1}^{40} \quad 1 \leq j \leq J-1.$$

Sort 20 ft containers according to a descending order of their weights, i.e.,

$$W_i^{20} \geq W_{i+1}^{20} \quad 1 \leq i \leq I-1.$$

Step 2: Pairing

Pair 20 ft containers sequentially to generate $Pair(n,1)$, $Pair(n,2)$ for $n = 1, \dots, N^{20}$ subject to load balancing constraints ($W_{Pair(n,1)}^{20} - W_{Pair(n,2)}^{20} \leq \bar{W}_d$).

Step 3: Matching

Match 20 ft container pairs with 40 ft containers sequentially to generate $Match(m,1)$, $Match(m,2)$, $Match(m,3)$ for $m=1, \dots, M$ subject to payload constraints $W_{Match(m,1)}^{20} + W_{Match(m,2)}^{20} \leq \bar{P} - W_{Match(m,3)}^{40}$

and center-of-gravity height constraints $W_{Match(m,1)}^{20} + W_{Match(m,2)}^{20} \geq aW_{Match(m,3)}^{40} - b$. If a 40 ft container cannot find a pair of 20 ft containers to match, move it to the unmatched 40 ft container set. If a 20 ft container pair is not matched by any 40 ft container, move it to the unmatched 20 ft container pair set.

Step 4: Assigning

Assign matched groups, unmatched 20 ft container pairs and unmatched 40 ft containers being placed on cars according to the different conditions described in the decision tree of Figure 8 and the following decision rules. We will further discuss the typical cases for assigning containers on cars. There are two mutually exclusive cases in terms of K as the number of cars and M as the number of the matching groups of 20 ft containers and 40 ft containers.

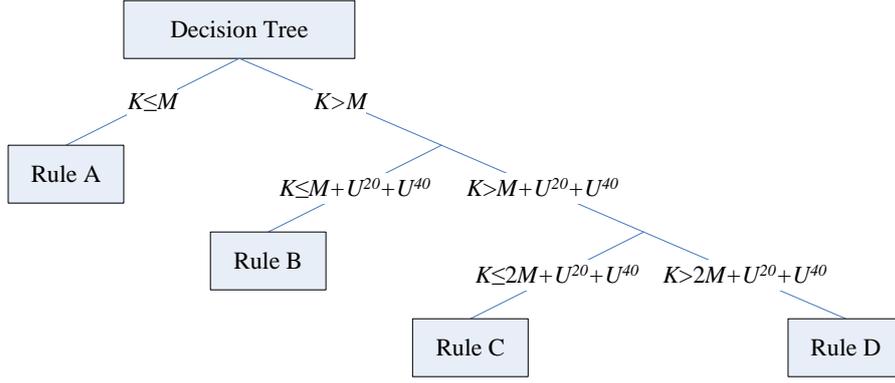


Figure 8. Container-to-car assignment decision tree

Rule A: Out of M matched groups of containers, select the K groups with the lowest loaded center-of-gravities to K cars separately. The loaded center-of-gravity of a matching group refers to the center of gravity of the car loaded with the matching group of containers.

Rule B: Assign M matched groups of containers to M out of K cars separately; select $(K-M)$ out of U^{20} unmatched 20 ft container pairs and U^{40} unmatched 40 ft containers to load on $(K-M)$ cars separately.

Rule C: Assign U^{20} unmatched 20 ft container pairs to U^{20} cars separately; assign U^{40} unmatched 40 ft containers to U^{40} cars separately; select $(K-M-U^{20}-U^{40})$ out of M matched groups of containers with the highest loaded center-of-gravities, and assign them to $2 \times (K-M-U^{20}-U^{40})$ cars, i.e., for each group of containers, assign the two 20 ft containers to one car and the 40 ft container to another car; assign $(2M+U^{20}+U^{40}-K)$ unsplit matching groups of containers to $(2M+U^{20}+U^{40}-K)$ cars.

Rule D: Assign U^{20} unmatched 20 ft container pairs to U^{20} cars separately; assign U^{40} unmatched 40 ft containers to U^{40} cars separately; assign M matched groups of containers to $2M$ cars, i.e., for each matched group of containers, assign the two 20 ft containers to one car and the 40 ft container to another car.

If $K \leq M$, then there are enough TEUs from matched containers (for the first objective). Thus, we focus on the second objective, and assign the matching groups with lower loaded center-of-gravities. This leads to Rule A. Otherwise (i.e. $K > M$), then there are $K-M$ cars left for unmatched containers. To increase the total TEUs loaded, we need to assign as many unmatched containers as possible to cars (using Rules B). In particular, in the case of $K > (M+U^{20}+U^{40})$, there are sufficient TEUs (from matched and unmatched containers) and the first objective cannot be further improved. As a result, we focus on the second objective by splitting some matching groups with higher loaded center-of-gravities. This leads to Rules C and D.

7.3. Improving solution

Based on an initial solution to the double-stack car loading problem, the second stage is designed to improve its solution quality by changing containers' positions. The following improvement algorithm is carried out for a certain number of iterations.

Step 1: From the current candidate solution, randomly and simultaneously select two 20 ft containers

and two 40 ft containers from all containers.

Step 2: Construct a neighbor solution by exchanging the positions of the two 20 ft containers and exchanging the positions of the two 40 ft containers. If this new neighbor solution is better than the candidate solution, then the current solution is replaced by this neighbor solution.

Note that containers staying on cars and being unloaded can be selected in this local search process. In our experiments, the number of iterations is set as $2I^2$ to permit sufficient solution coverage.

In the following illustrative example, the weights of nine 20 ft containers and four 40 ft containers are shown in Table 2 and Table 3, respectively.

Table 2. Weights of 20 ft containers.

20 ft container No.	1	2	3	4	5	6	7	8	9
Weight (tons)	30.0	28.2	26.8	25.5	24.1	13.0	10.6	8.7	6.3

Table 3. Weights of 40 ft containers.

40 ft container No.	1	2	3	4
Weight (tons)	30.2	26.6	20.7	15.3

The pairing algorithm from Stage I generate 20 ft container pairs shown in Table 4. Note that, 20 ft container No.5 is not paired, because the weight difference between containers No.5 and No.6 does not meet the load balancing constraint.

Table 4. 20 ft container pairs.

20 ft container pair No.	1	2	3	4
20 ft container No.	1, 2	3, 4	6, 7	8, 9

Shown in Table 5, the matching algorithm produces the following matching groups. There are four unmatched containers. Specifically, 40 ft container No. 1 is unmatched because it is too heavy to match with a 20 ft container pair subject to the payload constraint and the center-of-gravity height constraint. 20 ft container pair (8,9) is unmatched as there are not enough 40 ft containers to be matched. Recall that, 20 ft container No. 5 is left from the pairing process.

Table 5. Matching groups.

Group No.	40 ft container No.	20 ft container No.
1	2	6, 7
2	3	3, 4
3	4	1, 2

The assignment algorithm constructs car loading plans under different numbers of available cars, shown in Table 6.

Table 6. Loading plans under different number of available cars.

$K=3$			$K=4$			$K=5$			$K=7$			$K=9$		
Car No.	20' No.	40' No.	Car No.	20' No.	40' No.	Car No.	20' No.	40' No.	Car No.	20' No.	40' No.	Car No.	20' No.	40' No.

1	1, 2	4	1	6, 7	2	1	6, 7	2	1	8, 9	--	1	8, 9	--
2	3, 4	3	2	3, 4	3	2	3, 4	3	2	--	1	2	--	1
3	6, 7	2	3	1, 2	4	3	1, 2	4	3	6, 7	--	3	6, 7	--
			4	8, 9	--	4	8, 9	--	4	--	2	4	--	2
						5	--	1	5	3, 4	--	5	3, 4	--
									6	--	3	6	--	3
									7	1, 2	4	7	1, 2	--
												8	--	4
												9	--	--

For the initial solution under $K=3$ in Table 6, using the solution improvement algorithm, we can find an improved solution shown in Table 7. Specifically, the total TEUs loaded remain unchanged, but the center-of-gravity height decreases from 2,322.26 mm to 2,141.01 mm.

Table 7. Loading plan of an improved solution.

Car No.	20 ft container No.	40 ft container No.
1	1, 2	4
2	6, 7	3
3	3, 5	2

8. Computational experiments

To demonstrate the proposed lexicographic goal programming approach, the tabu search algorithm, and the two-stage heuristic algorithm for the double-stack car loading problem, we conduct computational experiments on a PC with a 2.0 GHz CPU and a 2 GB RAM.

First, two small-scale problems are used to compare the results of the three approaches. In the first example, five 40 ft containers and twelve 20 ft containers are to be loaded on five cars. In the second example, twelve 40 ft containers and twenty 20 ft containers are to be loaded on ten cars. The weights of the containers are randomly generated according to real-world container weight distributions. The optimal and heuristic solutions can be obtained using the lexicographic goal programming approach with GAMS (see Rosenthal (2008)) and the heuristic algorithm. The length of tabu list is set as 5 to solve the two examples. Table 8 compares the results of the three approaches, and Wd_{max} stands for the maximal weight difference of two 20 ft containers (in a single car) across all cars.

Table 8. The comparison of the results of three approaches.

Approach		Example 1				Example 2			
		TEUs	H_{max} (mm)	Wd_{max} (tons)	CPU Time (s)	TEUs	H_{max} (mm)	Wd_{max} (tons)	CPU Time (s)
Lexicographic Goal Programming Approach	Step1	20	—	—	0.523	40	—	—	1.836
	Step 2	20	2226.4	—	771	40	2182.5	—	7948
	Step 3	20	2226.4	3.71	29.78	40	2182.5	3.59	1010
	Total	20	2226.4	3.71	801	40	2182.5	3.59	8960
Tabu Search Algorithm		20	2226.4	3.71	0.015	40	2182.5	3.59	1.0
Two-Stage Heuristic Algorithm		20	2226.4	3.71	0.007	40	2182.5	3.59	0.015

In the lexicographic goal programming approach, the optimal center-of-gravity height is obtained by solving iterative mixed integer linear programming problems using the COINCBC solver. Figure 9 further details the iterative process of using the bisection algorithm in solving the first example, where the termination condition error bound ϵ is set to 1 mm.

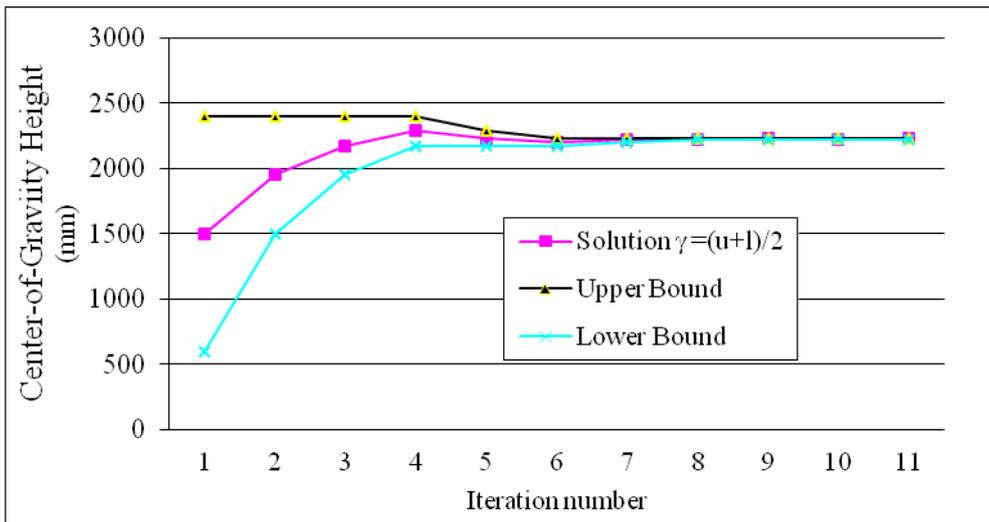


Figure 9. Iterative process of bisection algorithm

Table 8 indicates that, all of the three solution approaches obtain the same optimal solutions for these two examples. The lexicographic goal programming approach takes about 13 min to 2.5 hours, which is very time consuming compared to the two heuristic algorithms, even for small-scale instances. The optimization approach fails to solve a real-world instance with up to 300 containers and 100 double-stack cars within reasonable computing time limits. However, it is important to recognize the significance of the proposed optimization algorithms, since an optimal solution not only provides a yardstick for systematically evaluating ad-hoc heuristic algorithms but also generates an exact upper bound on the total operational cost savings attainable with alternative system design/improvement scenarios.

Table 8 also indicates that, the tabu search algorithm is less efficient than the two-stage heuristics algorithm for solving the small-scale double-stack car loading problems. To compare the computational

performance of the two proposed heuristic algorithms in real-world cases, we collected the real weights of 20 ft containers and 40 ft containers transported from Dahongmeng Station in Beijing to Yangpu Station in Shanghai. By using these data, we designed the following eight real-world cases.

- (1) 240 20 ft containers, 100 40 ft containers, 100 cars;
- (2) 240 20 ft containers, 100 40 ft containers, 90 cars;
- (3) 240 20 ft containers, 100 40 ft containers, 110 cars;
- (4) 240 20 ft containers, 100 40 ft containers, 130 cars;
- (5) 160 20 ft containers, 100 40 ft containers, 80 cars;
- (6) 160 20 ft containers, 100 40 ft containers, 70 cars;
- (7) 160 20 ft containers, 100 40 ft containers, 90 cars;
- (8) 160 20 ft containers, 100 40 ft containers, 110 cars.

The results of the two heuristic algorithms for solving the above eight cases are listed in Table 9. The length of tabu list is set as 10 to solve these eight cases.

Table 9. The results of the two heuristic algorithms for real-world cases.

Case No.	Two-Stage Heuristic Algorithm				Tabu Search Algorithm			
	TEUs	H_{max} (mm)	Wd_{max} (tons)	CPU time (s)	TEUs	H_{max} (mm)	Wd_{max} (tons)	CPU time (s)
(1)	400	2209.04	8.44	4.31	400	2209.54	9.94	9.08
(2)	360	2189.00	7.54	4.26	360	2189.70	9.84	8.45
(3)	420	2209.04	8.44	4.97	420	2209.64	9.98	9.55
(4)	440	2189.12	8.74	5.56	440	2209.44	10.00	10.4
(5)	320	2179.69	5.84	1.5	320	2187.14	9.92	3.11
(6)	280	2150.72	9.40	2.11	280	2159.91	9.54	2.81
(7)	340	2179.69	5.84	1.67	340	2192.56	9.78	3.23
(8)	360	2150.72	7.50	1.82	360	2190.11	9.92	3.58

Table 9 indicates that both of the two heuristic algorithms can obtain the close-to-optimal solutions of real-world double-stack car loading problems. Compared to tabu search algorithm, the two-stage heuristic algorithm can get better solutions in less computation times. So the two-stage heuristic algorithm is more effective and efficient than the tabu search algorithm in solving real-world double-stack car loading problems.

9. Conclusions

This paper presents a multi-objective model for the double-stack car loading problem, which specifically takes into account the operational productivity and safety concerns of the double-stack train service. A three-step lexicographic goal programming approach is designed to handle three objectives with different priorities. As the center-of-gravity height optimization problem is a generalized mixed integer linear-fractional programming problem, it is reformulated and further solved by a bisection algorithm. In summary, this research develops a systematic optimization solution procedure for the double-stack car loading problem, which has been further implemented and tested using the optimization software package GAMS.

To solve the real-world large-scale instances efficiently, a tabu search algorithm and a rule-based two-stage heuristic algorithm was designed. The tabu search algorithm is designed to generate an initial solution randomly and to improve the solutions by using 2-opt and tabu list techniques. The two-stage heuristic algorithm is designed to perform pairing, matching, assignment steps so as to rapidly construct an initial loading plan that satisfies a large number of constraints. At each step, the priority of different objectives is dynamically adjusted based on the given input and intermediate solution results. A local search algorithm is further used to improve the final solution quality. Both optimization and heuristic approaches are tested on two small-scale instances, and the two heuristic approaches can obtain the same optimal solutions while taking reasonable computational resources. The computational performance of the two proposed heuristic algorithm is also demonstrated by using several real-world cases. The results indicate that the two-stage heuristic algorithm is more effective and efficient than the tabu search algorithm in solving real-world double-stack car loading problems.

The future study will further take into account the destinations of the containers and the trains so that containers can be handled efficiently at following container transfer terminates. The priority of containers should also be addressed in the optimization process to meet shippers' requirements. In addition, alternative heuristics can be designed to take advantage of problem-specific characteristics in the double-stack car loading problem.

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References

Anderson, E., Haggstrom, J., Sima, M., Stichel, S., 2004. Assessment of train-overtaking risk due to strong cross-winds. Proceedings of the Institution of Mechanical Engineers, Part F: Journal of Rail and Rapid Transit 218: 213, 213-223.

Andersson, E., and Berg, M., 2001. Railway Systems and Rail Vehicles, Course Book, Part 2, Section 13.1.8, Kungliga Tekniska Hogskolan Railway Technology, Stockholm.

"APTA" American Public Transit Association Commuter Rail Executive Committee, 2007. APTA SS-M-015-06 Standard for Wheel Flange Angle for Passenger Equipment. The American Public Transportation Association, Washington, D.C.

"AREMA" The American Railway Engineering and Maintenance-of-Way Association, 2003. Practical Guide to Railway Engineering. Chapter 6, Railway Track Design, Lanham, MD.

"BNSF" Burlington Northern Santa Fe Railway, 2005. BNSF Intermodal Loading Guide: Loading Double Stack. Ft. Worth, TX.

Brandbury, W. M. S., Pottrill, P., Murtagh, V., 2003. Enhanced permissible speed on the West Coast mainline: investigations of wind overturning effects. In Proceedings of the 6th World Congress on Railway Research, Edinburgh, 476-485.

Cleon, L. M., and Jourdain, A., 2001. Protection of line LN5 against cross winds. In Proceedings of the 5th World Congress on Railway Research, Koln, Germany.

Dukkipati, R. V., and Amyot, J. R., 1988. Computer-aided simulation in railway dynamics. New York, NY. Marcel Dekker, Inc.

Freight Car America, Inc., 2011. The BethGon Aeroflo. Retrieved from <http://www.freightcaramerica.com/aluminum-bethgon-aeroflo.htm>.

Fujii, T., Maeda, T., Ishida, H., Imai, T., Tanemoto, K., Suzuki, M., 1999. Wind-induced accidents of trains/vehicles and their measures in Japan. Q. Rep. RTRI, 40(1), 50-55.

Gautier, P. E., Tielkes, T., Sourget, F., Allain, E., Grab, M., Heine, C., 2003. Strong wind risks in railways: the DEUFRAKO crosswind program. In Proceedings of the 6th World Congress on Railway Research, Edinburgh, 476-485.

Gawthorpe, R. G., 1994. Wind effects on ground transportation. J. Wind Engng Ind. Aerodynamics, 52, 73-92.

Holowaty, M. C., Cogswell, R. U., Moyer, N. E., 2004. Technical Monograph: Transportation Planning for the Richmond-Charlotte Railroad Corridor. Federal Railroad Administration US Department of Transportation, Washington, D.C. Volume II.

Ichoua, S., Gendreau, M., Potvin, J. Y., 2003. Vehicle dispatching with time-dependent travel times. European Journal of Operational Research 144(2), 379-396.

Imai, A., Sasaki, K., Nishimura, E., Papadimitriou, S., 2006. Multi-objective simultaneous stowage and load planning for a container ship with container rehandle in yard stacks. European Journal of Operational Research 171(2), 373-389.

Jahren, C. T. and Rolle, S. S., 1994. A computerized assignment algorithm for loading intermodal containers to double-stack railcars. Final report. University of Washington.

Jahren, C. T. and Rolle, S. S., Spurgeon, L. E., Palmer, R. N., Newman, R. R., Howland, D. L., 1995. Automatic assignment algorithms for loading double-stack railcars. Transportation Research Record 1511, 10-18.

Lippert, S., 1999. On Side Wind Stability of Trains, Kungliga Tekniska Hogskolan Railway Technology, Stockholm.

Lippert, S., Tengstrand, H., Andersson, E., Stichel, S., 2000. The effect of strong cross winds on rail vehicles. VDI Ber., 1586, 221.

Loumiet, J. R. and Jungbauer, W. G., 2005. Train Accident Reconstruction and FELA and Railroad Litigation. Tucson, Az. Lawyers and Judges Publishing Company, Inc.

Mancini, G., Cheli, R., Roberti, R., Diana, G., Cheli, F., Tomasini, G., Corradi, R., 2003. Cross-wind aerodynamic forces on rail vehicles – wind tunnel experimental tests and numerical dynamic analysis. In Proceedings of the 6th World Congress on Railway Research, Edinburgh, 513-522.

Matschke, G., Tilkes, T., Deeg, P., Schulte-Werning, B., Locher, P., Fermaud, C., Bohnenblust, H. , 2000. Effects of strong cross winds on high-speed trains – a risk assessment approach. In Proceedings of the International Conference on Probabilistic Safety Assessment and Management, Osaka, Japan.

Mongeau, M. and Bes, C., 2003. Optimization of aircraft container loading, IEEE Transactions on Aerospace and Electronic Systems, 39(1), 140-150.

Norfolk Southern, 1999. Norfolk Southern Intermodal Operations Manual: Loading and Securement: Stack Cars., Norfolk, VA.

Pacanovsky, D. L., Jahren, C. T., Palmer, R. N., Newman, R. R., and Howland, D., 1995. A decision support system to load containers to double-stack rail cars. Civil Engineering and Environmental Systems 11(4), 247-261.

Raidl, G. R., Kodydek, G., 1998. Genetic algorithms for the multiple container packing problem. Lecture Notes in Computer Science 1498, 875-884.

“RGS” Railway Group Standard GM/RT 2142., 1994. Resistance of Railway Vehicles to Overturning in Gales, Issue One, Safety and Standard Directorate Railtrack, Great Brittan.

Rosenthal, R. E., 2008. GAMS user’s guide, GAMS Development Corporation, Washington, DC, USA.

Schulte-Werning, B. and Matschke, G., 1997. Measures and strategies to minimize the effect of strong cross-winds on high-speed trains. In Proceedings of the 3rd Words Congress on Railway Research, Florence, Italy, E, 569-575.

Soak, S. M., Lee, S. W., Yeo, G. T., Jeon, M. G., 2008. An effective evolutionary algorithm for the multiple container packing problem. Progress in Natural Science 18(3), 337-344.

Talgo Inc., 2011. Talgo Technological Principals. Available from <http://www.talgoamerica.com/pdf/Talgo-Technological-Principals-Brochure.pdf> [Accessed 1 May 2011]

Tamiz, M., Jones, D., Romero, C., 1998. Goal programming for decision making: An overview of the

current state-of-the-art. *European Journal of Operational Research* 111(3), 569-581.

Union Pacific Railroad., 2010. *Intermodal Reports Database: Intermodal Ramp Policy Load Planning*. Omaha, NE.

“USDOT-FRA” U.S. Department of Transportation Federal Railroad Administration., 2002. *Track Safety Standards Compliance Manual*. Department of Transportation Office of Safety Assurance and Compliance, Washington D.C.

“USDOT-FRA” U.S. Department of Transportation Federal Railroad Administration., 2005. *Safe Placement of Train Cars: A Report*. Report to the Senate Committee on Commerce, Science and Transportation and the House Committee on Transportation and Infrastructure, Washington D.C.

“USDOT-FRA” U.S. Department of Transportation Federal Railroad Administration., 2011. *Track Safety Standards, Final Rule*. 49 CFR Part 213 Department of Transportation, Washington, D.C.